plate; V_0, jet velocity at a hole; V_δ, jet velocity at distance δ from the source; Y^2, dispersion; N_{Nu}, Nusselt number; N_{Re}, Reynolds number; and N_{Pr}, Prandtl number.

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A CIRCULAR TURBULENT JET IN A CROSSFLOW

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An integral method is proposed for calculating a circular turbulent jet propagating in a crossflow. The jet parameters obtained by a numerical method for different values of $\bar{q}$ were compared with experimental data. Satisfactory agreement between the sets of data was found.

The interaction and mixing of jets with a crossflow is a complex form of jet flow, and the study of the propagation of such jets is important for planning and designing equipment and devices in which mixing takes place. Several works by foreign and domestic authors have been devoted to the theoretical [1-5] and experimental [6-13] study of the laws of mixing and propagation of turbulent jets in a crossflow. As a rule, the theoretical studies [1-5] are based on integral methods, assume an increase in jet width, and make various other assumptions regarding the conditions of momentum conservation. Most of the investigations have focused on determining the jet axis, and only certain studies have examined laws of change in width, axial velocity, apparent additional mass, and other parameters.

It was shown in [5] that a jet propagating in a crossflow does not possess the property of similitude. This is evidenced first of all by the fact that, in the construction of lines of equal velocity, the cross sections of the jet change from a circular to a horseshoe shape. Such a change in jet development along its length leads to problems in analytically describing profiles of velocity, temperature, and concentration in its cross sections. In connection with this, it was proposed in [11] that the jet flow region be broken down into three sections, within each of which the flow could be assumed to possess the property of similitude. Meanwhile, according to the data in [11], the determining change in the cross-sectional shape of the jet occurs in the initial section. However, this was not confirmed by the experiment in [7]. In the present work, we attempt to analytically determine the above jet parameters within a broad range of values of the hydrodynamic parameter $\bar{q}$ ($4 \leq \bar{q} \leq 400$). The method of calculation is based on several assumptions: the jet axis is the locus of the points where the velocity for each section normal to the direction of the jet is maximal; the jet is bounded by the surface on which the excess velocity in the direction of the axis decreases to less than a specified low value.

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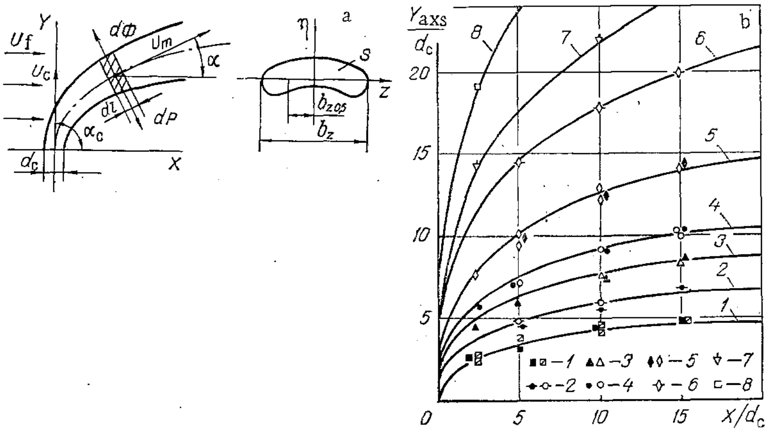

Fig. 1. Drawing (a) and trajectory (b) of jets: 1-8) $\bar{q}$ = 4.75; 10; 16; 25; 50; 125; 200; 400, respectively; clear points – [9]; dark points – [10]; curves – calc.

We will examine an isothermal jet of incompressible fluid issuing from a circular hole normal to the crossflow. The propagation of the jet in the uniform crossflow is described in a rectangular system of coordinates, X, Y, Z, with its origin at the center of the hole from which the jet escapes (Fig. 1a). The equation of equilibrium of the forces acting on an isolated element of the jet projected onto a normal to the jet axis has the form [1]

$$dP = -d\Phi, \tag{1}$$

where dP is the projection onto the normal of the force from the pressure field acting on the side of the element; dΦ is the centrifugal force which arises with the motion of the element over the curvilinear trajectory. It can be described thus:

$$d\Phi = \frac{\rho U^2 S}{R}\, dl, \tag{2}$$

where U is the mean mass velocity, determined from the formula

$$U = \frac{1}{S} \int\limits_S \int u dS.$$

The force of the pressure dP is described by analogy with flow about a solid and is proportional to the characteristic frontal area and the velocity head, which is calculated from the projection of the crossflow velocity onto the axis normal:

$$dP = C_n \frac{\rho_f U_f^2}{2} \sin^2\alpha b_z dl, \tag{3}$$

where C_n is the drag coefficient, dependent on the dynamic heads of the jet and flow.

Allowing for (2) and (3), Eq. (1) takes the form

$$C_n \frac{\rho_f U_f^2}{2} \sin^2\alpha b_z dl = -\frac{\rho U^2 S}{R} dl. \tag{4}$$

Equation (4) is solved using various assumptions regarding conservation of momentum. Analysis of calculated results shows that assumption of conservation of the momentum projection along the Y axis or constancy of excess momentum along the jet does not lead to satisfactory agreement with the empirical data, especially at low $\bar{q}$. At high $\bar{q}$, the above assumptions give results close to the experimental findings. Thus, we will use the assumption of constant total momentum along the jet [4], which leads to the best agreement between the calculated and experimental data:

$$\rho U^2 S = \rho_c U_c^2 S_c = \text{const}. \tag{5}$$

Allowing for the assumption (5), Eq. (4) is written:

$$C_n \bar{R} \bar{b}_z \sin^2\alpha = -\frac{\pi \bar{q}}{2}. \tag{6}$$

Introducing the well-known relations

$$\bar{R} = \frac{(1+\bar{Y}'^2)^{3/2}}{\bar{Y}''}, \quad \sin\alpha = \frac{\bar{Y}'}{\sqrt{1+\bar{Y}'^2}},$$

we reduce Eq. (6) to the following form:

$$C_n \frac{(1+\bar{Y}'^2)^{3/2}}{\bar{Y}''} \frac{\bar{Y}'^2}{1+\bar{Y}'^2} \bar{b}_z = -\frac{\pi\bar{q}}{2}. \tag{7}$$

We determined the change in jet width $\bar{b}_z$ along the jet from the width of the submerged jet $\bar{b}_s$ using an empirical correction accounting for the effect of the crossflow:

$$\bar{b}_z = \bar{b}_s \left(1 + 1.6\bar{q}^{0.55} \frac{\pi/2 - \operatorname{arctg} \bar{Y}'}{\bar{l}}\right). \tag{8}$$

The following relation is proposed for finding the coefficient of expansion of the submerged jet:

$$C = \frac{b_s}{2l_1} = C_{\bar{\rho}=1}\left[1.634 - 0.317\left(\lg 100 \frac{\rho_m}{\rho_H}\right)\right]. \tag{9}$$

It follows from (9) that C decreases with increasing distance from the nozzle edge for a jet with a density at the nozzle outlet ρ_c which is less than ambient density ρ_H, while it increases for a jet with $\rho_c > \rho_H$. The coefficient of expansion remains constant for submerged jets of an incompressible fluid ($\rho_c = \rho_H = \rho_m$): $C = C_{\bar{\rho}=1} = 0.167$. The distance from the pole of the jet to the section in question $l_1 = l_{po} + l$ increases with the jet density, since the distance from the nozzle edge to the pole l_{po} increases. For a submerged jet of an incompressible fluid, $l_{po} = 3d_c$ [13]. Substituting empirical relations (8) and (9) into (7) and effecting certain transformations using the new variable $p = \bar{y}'$, we obtain

$$p' = -\frac{4CC_n}{\pi\bar{q}}(\bar{l}_{po} + \bar{l})\left(1 + 1.6\bar{q}^{0.55} \frac{\pi/2 - \operatorname{arctg} p}{\bar{l}}\right) p^2 \sqrt{1+p^2}, \tag{10}$$

where $\bar{l} = \int_0^{\bar{x}} \sqrt{1+p^2}\, d\bar{X}$. The following empirical relation is proposed for the drag coefficient C_n on the basis of generalization of the empirical data on jet axis trajectory:

$$C_n = 1.05\bar{q}^{0.1}. \tag{11}$$

Equation (10) was solved on a computer by the finite differences method (Euler's broken line) using second-order boundary conditions:

$$\text{at } X = 0: \; Y = 0, \; \frac{dY}{dX} = \operatorname{tg}\frac{\pi}{2}.$$

The relative error of the calculations was no greater than 1% with a mesh corresponding to 0.01 nozzle diameter. The calculations gave us values of jet width $\bar{b}_z$ and jet axis coordinates $\bar{X}$ and $\bar{Y}$. Figure 1b shows the trajectory of the jet axis for different values of $\bar{q}$. It is apparent from Fig. 1b that the calculations agree with the experimental data. The change in width along the jet $\bar{l}$ is shown in Fig. 2. The rate of increase in width at small distances from the nozzle ($\bar{l}$ = const) is greater at low values of $\bar{q}$, while the change in width at substantial distances from the nozzle is more substantial for jets with high values of $\bar{q}$. This pattern of increase in jet width is explained by the fact that, at low values of $\bar{q}$, velocities at the nozzle outlet and the static pressure around the jet are redistributed under the influence of the crossflow. Interaction of the jet with the crossflow causes an increase in static pressure in front of the nozzle and a decrease in exhaust velocity in the front part of the jet (orificing of the top part of the nozzle), while an increase in velocity in the lateral regions of the jet at the nozzle edge indicates the existence of external rarefaction regions on the sides of the rear part of the jet. A pair of eddies of opposite rotation is thus created close to the nozzle, and the greatest rarefaction is seen at the sites where the eddies are originated, i.e., at the sides of the nozzle. This phenomenon leads to stretching of the jet in the transverse direction and, thus, to an increase in jet width. The increase in width decreases markedly with increasing distance from the nozzle, since – as is evident from Fig. 1b – the jet becomes a cocurrent stream fairly rapidly. With high $\bar{q}$, the momentum of the jet near the nozzle is significantly higher than that of the crossflow, so the presence of rarefaction regions at the sides of the nozzle does not cause the expansion of the jet near the nozzle seen with low values of $\bar{q}$. Here, the increase in jet width is not greatly different from that of the submerged jet. However, due to the decreasing velocity of the jet with increasing distance from the nozzle, the sides of the jet come to be influenced to the same degree by the crossflow as in the case of jets with low $\bar{q}$, and the rate of increase in width increases.

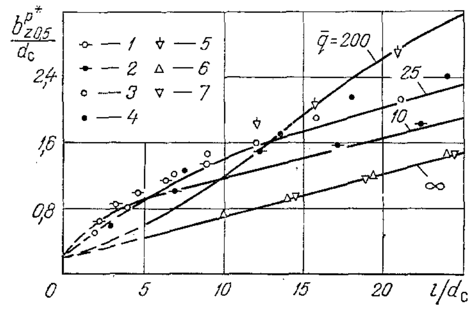

Fig. 2. Jet half-width: 1, 2) $\bar{q}$ = 10; 3, 4) 25; 5) 200; 6, 7) $\bar{q} = \infty$ (1, 3 – [3]; 6, 7 – [13]; 2, 4, 5 – our data; curves – calculated).

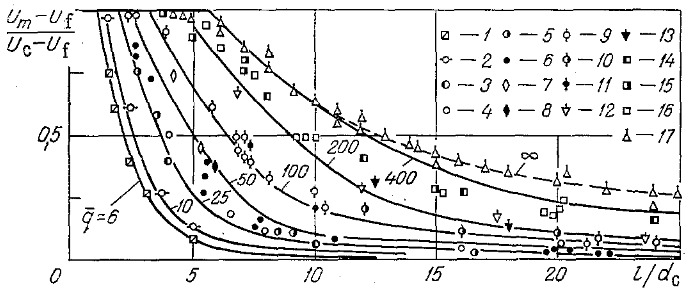

Fig. 3. Axial velocity of jet: 1) $\bar{q}$ = 6; 2) 10; 3-6) 25; 7, 8) 50; 9-11) 100; 12, 13) 200; 14-16) 400; 17) $\bar{q} = \infty$ (1, 2 – [3]; 5, 7, 11 – [8]; 4, 6, 8, 9, 12, 15, 16 – [6]; 14 – [2]; 17 – [13]; 3, 10, 13 – our data; curves – calculated).

Figure 3 shows the change in axial relative excess velocity along the jet for different values of $\bar{q}$. It follows from the data that a decrease in $\bar{q}$ is accompanied by a reduction in the size of the initial section and an increase in the rate of decrease in axial velocity. These factors lead to an increase in the range of the jet with an increase in $\bar{q}$. By range, we mean the distance from the nozzle to the point on the jet axis where the excess velocity is 10% of the excess velocity at the nozzle outlet. It follows from Fig. 3 that at $\bar{q}$ = 6 the range is equal to five nozzle diameters (l/d_c = 5). This effect is connected with the fact that at low $\bar{q}$ the jet rapidly loses its individuality as a result of mixing of the working body of the jet with the crossflow. The experimental data shown in Fig. 3 is very satisfactorily described by the empirical formula

$$\frac{u_m - U_f}{U_c - U_f} = \frac{\left[1 - \exp\left(-\frac{0.5\bar{q}^{0.9}}{\bar{l}^2}\right)\right]\sqrt{\bar{q}} + \cos\alpha - 1}{\sqrt{\bar{q}} - 1}. \tag{12}$$

The results of calculation of axial velocity along the length of a submerged jet in [13] are shown by the dashed curve in Fig. 3. The apparent additional mass of the jet was determined with the assumption of conservation of momentum (5), which was described in the form

$$GU = G_c U_c. \tag{13}$$

It follows from (13) that finding the apparent additional mass requires knowledge of the change in the mean mass velocity U along the jet for different values of $\bar{q}$. Based on physical considerations and the experimental data, the following empirical relation is proposed between mean mass velocity U and axial velocity u_m:

$$\frac{U - U_f \cos\alpha}{u_m - U_f \cos\alpha} = 1 - \exp\left(-0.03\frac{\bar{q}}{\bar{l}^2} + \ln 0.75\right). \tag{14}$$

Then the expression for the relative apparent additional mass can be written thus:

$$\frac{G - G_c}{G_c} = \frac{1}{\left(\frac{U - U_f \cos\alpha}{u_m - U_f \cos\alpha}\right)\left[1 - \exp\left(-\frac{0.5\bar{q}^{0.9}}{\bar{l}^2}\right)\right] + \frac{\cos\alpha}{\sqrt{\bar{q}}}} - 1. \tag{15}$$

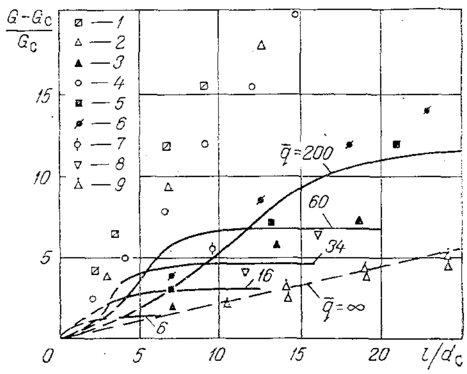

Fig. 4. Apparent additional mass of a jet: 1) $\bar{q}$ = 6; 2, 3) 16; 4) 25; 5) 34; 6) 60; 7) 100; 8) 250; 9) $\bar{q} = \infty$ (1, 2, 4 – [3]; 3, 5, 6 – [7]; 9 – [13]; 7, 8 – our data; curves – calc.).

Figure 4 shows results of calculations with Eqs. (15) and experimental data from [3, 7] on the apparent additional mass of a jet. The experiments in these works, conducted at the same values of $\bar{q}$, came up with quite different results. For example, for $\bar{q}$ = 16 and $\bar{l}$ = 7, the apparent additional mass is 9.5 according to [3] and 2 according to [7]. These works also show a qualitative difference in the character of change in apparent additional mass in relation to $\bar{q}$. With $\bar{l}$ = const and a decrease in $\bar{q}$, the apparent additional mass of the jet increases according to [3] and decreases according to [7]. The differences in apparent additional mass values in [3] and [7] complicates comparison of these sets of data with the calculated results. However, a qualitative explanation of the calculated data can be found if we follow the behavior of jet width (see Fig. 2) and the decrease in the axial velocity of the jet (see Fig. 3). The presence of the pair of eddies at the sides of the nozzle in the case of low $\bar{q}$ leads to intensive suction of air into the jet from the region near the nozzle [12]. The ejection capacity of these eddies is substantially greater than the turbulent mixing of the submerged jet, due to the high degree of turbulence in the vicinity of the eddies. However, with increasing distance from the nozzle, dissipation of the energy of the eddies and decrease of the velocity of the jet result in a decrease in the ejection capacity of the latter, and the increase in apparent additional mass becomes slight. With high values of $\bar{q}$, the reduction in axial velocity and change in jet cross-sectional dimensions at short distances from the nozzle differ little from the laws of development of the submerged jet, so that the jet has nearly the same ejection capacity. With increasing distance from the nozzle, the mass increment gradient increases as the jet is developed in the crossflow, because the velocities of the jet and crossflow become comparable and the laws of increase in apparent additional mass become the same as for jets with low values of $\bar{q}$ at short distances from the nozzle.

It should be noted in conclusion that, for jets with low $\bar{q}$, width (Fig. 2) and apparent additional mass (Fig. 4) increase most rapidly at short distances from the nozzle. Such a mutual change in these parameters is connected with the fact that weak jets flowing normal to a crossflow of comparable velocity become cocurrent streams not far from the nozzle. Conversely, for jets with high values of $\bar{q}$, the increase in width and apparent additional mass and decrease in axial velocity close to the nozzle occur in qualitatively the same way as with a submerged jet. Here, the jet is hardly deflected from its initial direction by the crossflow. Only with increasing distance from the nozzle, as the jet loses velocity, does it begin to bend; width and apparent additional mass begin to increase more rapidly.

NOTATION

d_c, nozzle diameter, m; l, distance along jet axis, m; $\bar{l}$, ratio of distance along jet axis to nozzle diameter; Y_{axs}, jet axis coordinate, m; U_c, U_f, initial velocities of jet and flow, respectively, m/sec; ρ_c, ρ_f, densities of jet and flows, kgf/m^3; $\bar{q} = \rho_c U_c^2/\rho_f U_f^2$, ratio of velocity heads of jet and flow.

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INTENSITY OF HEAT EXCHANGE NEAR A STAGNATION POINT IN THE COURSE OF PERIODIC VARIATION OF THE SURFACE TEMPERATURE

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We present the solution of the nonstationary problem of heat exchange near a stagnation point of the flow at a barrier, represented by a spherical surface, when the surface temperature of the body is periodically varied.

In practice, one often encounters the case of heat exchange between a flow and a barrier when the periodic variation of the flow parameters is close to steplike. An example is the application of obturator devices which periodically cut off the flow from a heated barrier [1]. The burning of an electric arc in a linear plasmotron with short electrodes represents blowing and ignition at high frequency [2], which leads to a periodic change of temperature of the generated jet. There are situations when heat is exchanged in the course of periodic variation of the surface temperature.

The results of investigation of the characteristics of a heat exchange with a barrier on a model of the process, represented by a steplike periodic variation of the surface temperature between arbitrary values, are given below. The full period is divided into two different time intervals during which the temperature is constant. The change of temperature takes place at the ends of the time intervals.

In the solution of this problem, we have made a preliminary study of a nonstationary heat exchange at a stagnation point of a sphere after a stepwise change of temperature of the surface (or of the flow). As in [3, 4], the flow is assumed to be subsonic, laminar, and stationary, and the properties of the liquid are assumed constant.

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